25.1 The side BC of ΔABC is extended beyond C to D so that $CD = BC$. The side CA is extended beyond A to E so that $AE = 2 \cdot CA$. Prove that if $AD = BE$ then the triangle ABC is right-angled.

25.2 Show that $7^{37} + 13^{37} + 19^{37}$ is divisible by 39.

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26. Let A and E be opposite vertices of an octagon. A frog starts at vertex A. From any vertex except E, it jumps to one of the two adjacent vertices. When it reaches E, it stops. Let a_n be the number of distinct paths of exactly n jumps ending at E. Prove that $a_{2n-1} = 0, a_{2n} = \frac{(2+\sqrt{2})^{n-1} - (2-\sqrt{2})^{n-1}}{\sqrt{2}}$.

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27.1 For $a, b, c \in \mathbb{R}^+$, prove that $\frac{a^3}{a^2+ab+b^2} + \frac{b^3}{b^2+bc+c^2} + \frac{c^3}{c^2+ca+a^2} \geq \frac{a+b+c}{3}$.

27.2 Determine all functions $f: \mathbb{R} \to \mathbb{R}$ such that $f(x+y) = f(x-y)$ for $\forall x, y \in \mathbb{R}$.

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28. For $x, y \in \mathbb{Z}$, prove that $y^2 + 4 = x^5$ has no solutions.

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29. The point O is situated inside the parallelogram $ABCD$ such that $\angle AOB + \angle COD = 180°$. Prove that $\angle OBC = \angle ODC$.

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30. There are an even number of people at a party. Each person has an even number of friends at the party. Remember that friendship is mutual; a is friend with b and b is also friend with a. Let p be any person at the party and prove that there is a person q (where is $p \neq q$) who has an even number of mutual friends with p.

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31. Prove that the equation $4(x_1^4 + x_2^4 + \cdots + x_{14}^4) = 7(x_1^3 + x_2^3 + \cdots + x_{14}^3)$ has no solutions in positive integers.

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24. Let a, b, c, d be distinct real numbers. Show that the equations

$$ax^2 + (b + d)x + c = 0$$

$$bx^2 + (c + d)x + a = 0$$

$$cx^2 + (a + d)x + b = 0$$

have a common solution if and only if $a + b + c + d = 0$.

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